



WAR with Auction Rounds

Assumptions

Since:

- 1 The starting state of each player is known to all other players
- 2 Every player knows what all other players played

We may assume:

Perfect Information Principle

Since every player is capable of “counting cards”, every player is theoretically capable of knowing the cards in every other player i 's hand (H_i) and wins pile (W_i).

WAR as a Repeated Static Game

WAR can be represented as a repeated static game; that is, the outcome of a player's move depends on the other players' moves, but state is maintained between moves to formulate the outcome of game.

Problem Statement

You already know how WAR works.

Problem Statement

You already know how WAR works.

Given a game of WAR with N players, devise a **strategy** to “do well” at WAR.

Dead Simple Strategy: DummiePlayer (DP)

DummiePlayer_{*i*} (DP):

- To play a normal move, select randomly from H_i
- To play a tie, choose 4 cards randomly from H_i and discard the first 3, play the last

Dead Simple Strategy: DummiePlayer (DP)

DummiePlayer_{*i*} (DP):

- To play a normal move, select randomly from H_i
- To play a tie, choose 4 cards randomly from H_i and discard the first 3, play the last

How well does DP do? When playing against N other DP's, a DP will have a $\frac{1}{N}$ chance of winning.

An Intelligent Player: SimpleMindedPlayer (SMP)

SimpleMindedPlayer_i (SMP):

- To determine a normal play:
 - 1 Compute $S = \{H_{ic} \mid \forall j, d [H_{ic} > H_{jd}]\}$.
 - 2 If $S \neq \emptyset$, play $\min(S)$.
 - 3 Otherwise if, $|H_i| \geq 5$, $\exists j, c [H_{ic} = \max(H_j) \wedge \max_2(H_i) \geq \max_2(H_j)]$, then play the corresponding H_{ic} .
 - 4 Otherwise, play $\min(H_i)$.
- To play a tie, follow strategy similar to above, but remove 3 smallest cards from hand first to be discarded.

Example: SMP Move Computation ($S \neq \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

Example: SMP Move Computation ($S \neq \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

Example: SMP Move Computation ($S \neq \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \{10, 10, Q\}$.

Example: SMP Move Computation ($S \neq \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \{10, 10, Q\}$.
- 2 $S \neq \emptyset$, so we play $\min(S) = 10$.

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

1 Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \emptyset$.

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \emptyset$.
- 2 $S = \emptyset$, so we cannot play from S .

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d [H_{ic} > H_{jd}]\} = \emptyset$.
- 2 $S = \emptyset$, so we cannot play from S .
- 3 See if we can play high:

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d [H_{ic} > H_{jd}]\} = \emptyset$.
- 2 $S = \emptyset$, so we cannot play from S .
- 3 See if we can play high:
 - 1 $|H_i| \geq 5$? Yes.

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d[H_{ic} > H_{jd}]\} = \emptyset$.
- 2 $S = \emptyset$, so we cannot play from S .
- 3 See if we can play high:
 - 1 $|H_i| \geq 5$? Yes.
 - 2 $\exists j, c [H_{ic} = \max(H_j) \wedge \max_2(H_i) \geq \max_2(H_j)]$? Yes, $j = 2$, $H_{jc} = Q$.

Example: SMP Move Computation ($S = \emptyset$)

Player 1 is a SMP. Players 2, 3, 4 are opponents of unknown strategy.

$$H_1 = \{8, 9, 10, 10, Q\}$$

$$H_2 = \{4, 5, 7, 7, 9, Q\}$$

$$H_3 = \{2, 3\}$$

$$H_4 = \{5, 9\}$$

- 1 Compute $S = \{H_{ic} \mid \forall j, d [H_{ic} > H_{jd}]\} = \emptyset$.
- 2 $S = \emptyset$, so we cannot play from S .
- 3 See if we can play high:
 - 1 $|H_i| \geq 5$? Yes.
 - 2 $\exists j, c [H_{ic} = \max(H_j) \wedge \max_2(H_i) \geq \max_2(H_j)]$? Yes, $j = 2$, $H_{jc} = Q$.
 - 3 Play Q.

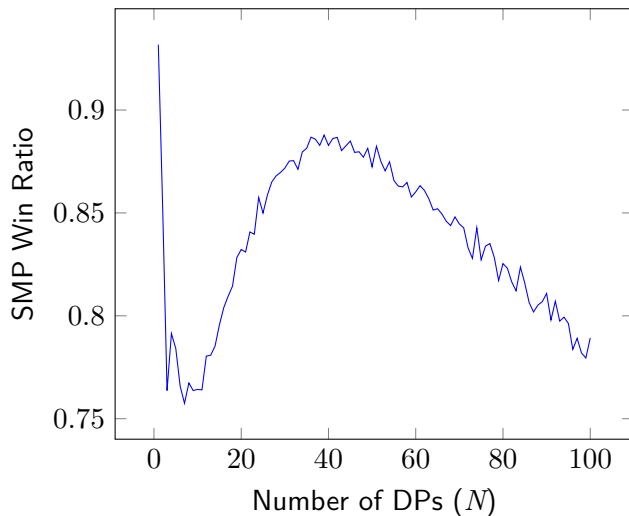
How well does a SMP do against N other SMPs? A SMP will always die against a fellow SMP, and the game will result with no winners. So an SMP stands a 0% chance here.

How well does a SMP do against N other SMPs? A SMP will always die against a fellow SMP, and the game will result with no winners. So an SMP stands a 0% chance here.

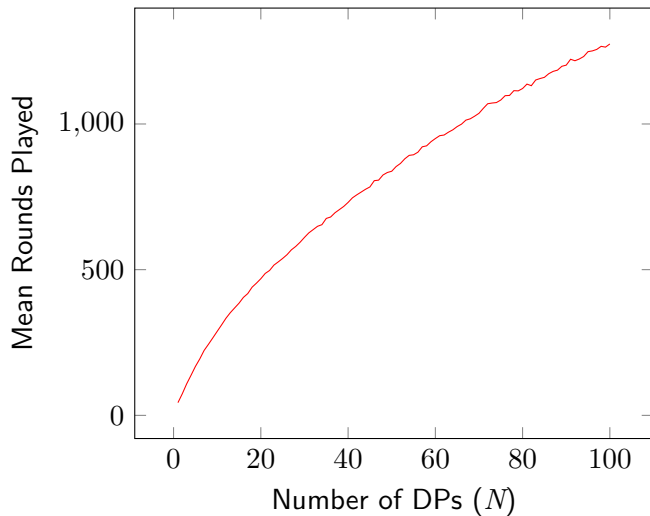
How well does a SMP do against N DPs? Too hard to figure out mathematically, so we wrote a computer simulation to answer this.

- <https://github.com/psattiza/war>
- 100,000 simulated games
- A single SMP battles N DPs, for all $N \in \{1, \dots, 100\}$.

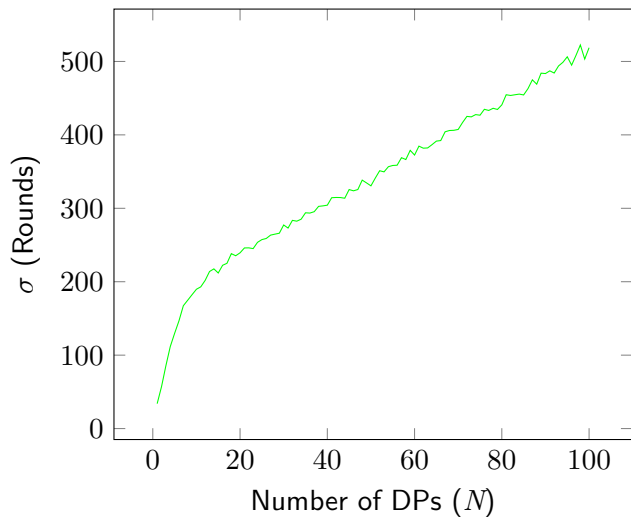
SMP vs. N DPs: Win Ratio



SMP vs. N DPs: Mean Rounds Played



SMP vs. N DPs: Variance of Rounds Played



- SMP does not take into account W_i for either themselves, or other players. Design a CMP (ComplexMindedPlayer) which adds in factors from W_i .

- SMP does not take into account W_i for either themselves, or other players. Design a CMP (ComplexMindedPlayer) which adds in factors from W_i .
- When an SMP determines it cannot win, it chooses it's lowest card, which may have a high probability of causing a tie against DPs. Design a player good for defeating large quantities of DPs that takes this into account.

- SMP does not take into account W_i for either themselves, or other players. Design a CMP (ComplexMindedPlayer) which adds in factors from W_i .
- When an SMP determines it cannot win, it chooses it's lowest card, which may have a high probability of causing a tie against DPs. Design a player good for defeating large quantities of DPs that takes this into account.
- Throw the game at a ML algorithm and see what happens?

- SMP does not take into account W_i for either themselves, or other players. Design a CMP (ComplexMindedPlayer) which adds in factors from W_i .
- When an SMP determines it cannot win, it chooses it's lowest card, which may have a high probability of causing a tie against DPs. Design a player good for defeating large quantities of DPs that takes this into account.
- Throw the game at a ML algorithm and see what happens?
- More simulation runs!

Questions?