WAR with Auction Rounds

Assumptions

Since:

1. The starting state of each player is known to all other players
2. Every player knows what all other players played

We may assume:

Perfect Information Principle

Since every player is capable of "counting cards", every player is theoretically capable of knowing the cards in every other player i's hand (H_i) and wins pile (W_i).

WAR as a Repeated Static Game

WAR can be represented as a repeated static game; that is, the outcome of a player's move depends on the other players' moves, but state is maintained between moves to formulate the outcome of game.

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Given a game of WAR with N players, devise a **strategy** to “do well” at WAR.

Dead Simple Strategy: DummiePlayer (DP)

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- To play a normal move, select randomly from H_i
- To play a tie, choose 4 cards randomly from H_i and discard the first 3, play the last

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How well does DP do? When playing against N other DP's, a DP will have a $\frac{1}{N}$ chance of winning.

An Intelligent Player: SimpleMindedPlayer (SMP)

SimpleMindedPlayer$i$ (SMP):

- To determine a normal play:
  1. Compute $S = \{H_{ic} \mid \forall j, d\,[H_{ic} > H_{jd}]\}$.
  2. If $S \neq \emptyset$, play $\min(S)$.
  3. Otherwise if, $|H_i| \geqslant 5$, $\exists j, c\,[H_{ic} = \max(H_j) \wedge \max_2(H_i) \geqslant \max_2(H_j)]$, then play the corresponding H_{ic}.
  4. Otherwise, play $\min(H_i)$.
- To play a tie, follow strategy similar to above, but remove 3 smallest cards from hand first to be discarded.

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   3. Play Q.

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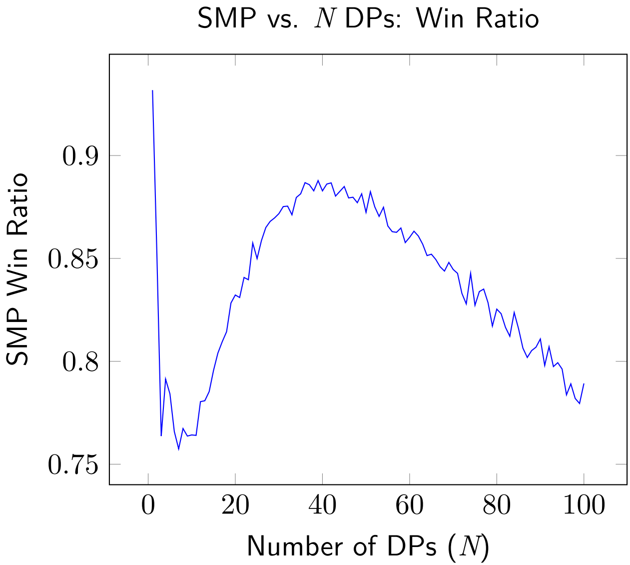
How well does a SMP do against N DPs? Too hard to figure out mathematically, so we wrote a computer simulation to answer this.

Computer Simulation

- `https://github.com/psattiza/war`
- 100,000 simulated games
- A single SMP battles N DPs, for all $N \in \{1, \ldots, 100\}$.

Computer Simulation



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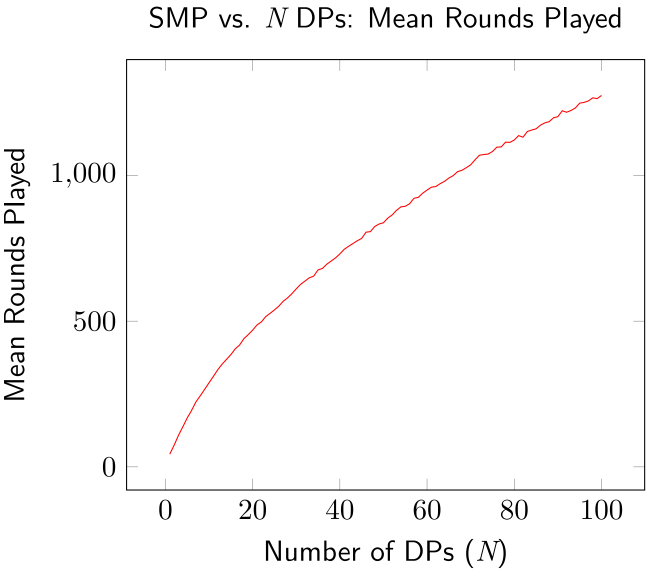

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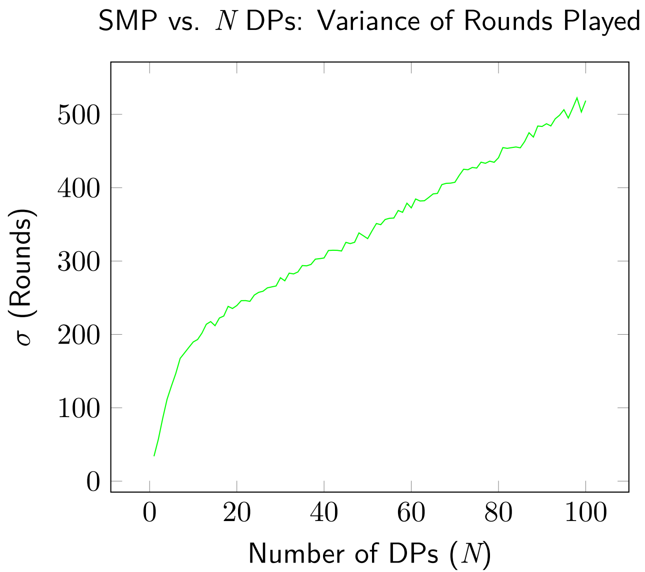

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- Throw the game at a ML algorithm and see what happens?
- More simulation runs!

Questions?